Phylogenetic Support

Statistical Testing of Trees

Finlay Maguire March 27, 2018

FCS, Dalhousie

- 1. Introduction
- 2. Evolutionary Model Testing
- 3. Branch Support Testing
- 4. Comparing Trees
- 5. Conclusion

Introduction

Phylogenies are hypotheses

Cid



- Does another model of sequence evolution fit the data better?
- How well supported are individual branches in a tree?
- Does another tree explain the data better?

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- Inappropriate inference

Saturation





Misleading Signal: Hidden Paralogy/Incomplete Sampling



[Leonard, 2010]

Misleading Signal: Horizontal Gene Transfer



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Ask for a tree get a tree.



Tree not always correct paradigm

Ask for a tree get a tree.



Reanalysis of [Marwick, 2012] from http://phylonetworks.blogspot.ca/2013/02/

Evolutionary Model Testing

Sequence Evolution Models



http: //carrot.mcb.uconn.edu/~olgazh/bioinf2010/class24.html

Sequence Evolution Models

	HIV-W _m	HIV-W _m +F	HIV-Bm	HIV-B _m +F	REV-1 step	JTT+F	лт	WAG+F	MtMAM+F	rtREV	mtREV 24+F	WAG	Dayhoff+F	rtREV+F	Dayhoff	Equal Input	mtREV 24	mtMAM	REV
HIV-W _m	0	45	44	46	47	46	47	47	47	46	47	47	47	47	47	47	47	47	47
HIV-Wm+F	1	0	45	46	46	46	46	47	47	47	47	47	47	47	47	47	47	47	47
HIV-B _m	0	1	0	15	43	30	39	43	46	46	46	46	46	47	47	47	47	47	47
HIV-B _m +F	0	0	15	0	43	37	40	44	47	46	47	46	47	47	47	47	47	47	47
REV-1 step	0	1	4	4	0	6	6	11	31	32	22	14	17	24	28	35	41	43	47
JTT+F	0	0	8	5	40	0	28	47	46	46	47	47	47	47	47	47	47	47	47
лт	0	0	3	3	38	4	0	35	44	46	45	47	47	46	47	47	47	47	47
WAG+F	0	0	3	1	34	0	5	0	43	44	43	39	42	46	47	47	47	47	47
MtMAM+F	0	0	0	0	16	0	0	2	0	14	2	6	4	7	12	31	47	47	46
rtREV	0	0	0	0	12	0	1	2	29	0	8	1	3	3	4	39	47	47	47
MtREV 24+F	0	0	0	0	18	0	1	1	41	37	0	7	7	22	25	47	47	47	47
WAG	0	0	0	1	29	0	0	2	40	45	35	0	30	39	43	46	47	47	47
Dayhoff+F	0	0	0	0	26	0	0	0	39	43	29	8	0	36	43	46	47	47	47
rtREV+F	0	0	0	0	19	0	0	0	35	41	20	2	1	0	20	46	47	47	47
Dayhoff	0	0	0	0	18	0	0	0	32	39	17	0	1	17	0	44	47	47	47
Equal Input	0	0	0	0	11	0	0	0	14	2	0	1	0	1	2	0	41	46	47
mtREV 24	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	4	0	43	45
mtMAM	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	1	0	44
REV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	0

¹Models are arranged by decreasing rank performance (see Table 2) doi:10.1371/journal.pone.0000503.t003

[Nickle et al., 2007]

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- Wrong tree support values

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- Limitations: nested models (i.e. hLRT), order matters, no regularisation

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- Decision Theory (DT) risk minimisation approach.

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- If the model is reasonable then data simulated under should resemble the empirical data

Branch Support Testing

Bootstrapping in General





Bootstrapping Phylogenies





Slide from Joe Felsenstein

(and so on)

Bootstrapping Phylogenies



Bootstrapping Phylogenies

The majority-rule consensus tree



Combining the results



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- Goal to simulate an infinite population (number of alignment columns)

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- Only answers to what extent does input data support a given part of the tree

- Simulate data sets of this size assuming the estimate of the tree is the truth
- Key for many more sophisticated tests.
- · Can be used to generate *p*-values, but non-trivial

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- Ultrafast Bootstraps (UFBoot)



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- aBayes:
- $P(T_c \mid X) = \frac{P(X|T_c)P(T_c)}{\sum_i^2 = 0P(X||T_i)P(T_i)}$ with flat prior $P(T_0) = P(T_1) = P(T_2)$

Comparing Trees

How to compare competing hypotheses?



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Simplistic Comparison




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- 40 out of 60 *p* = 0.0124
- 400 out of 600 $p = 2.3 * 10^{-16}$



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- $\sigma^2 = 15.22$
- $t = \frac{\mu}{\sigma^2} * \sqrt{N} = 0.148$
- therefore: p = 0.888 under 5*d.f.*

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- Why can't we just use χ^2 to get a critical value for δ ?
- Tree space is difficult.

- Many avenues:
- Non-parametric bootstrapping
- Parametric bootstrapping
- Related approaches.

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- Can't handle multiple comparisons.

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- Sensitive to model misspecification.

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- Model selection typically relies on multiple metrics
- Bootstrapping is a slow, biased but conservative way to estimate the support for a given branch in your tree.
- Likelihood Testing is powerful but must be used with care.
- Comparing trees directly is non-trivial due to tree-space.

Questions?

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